Journal of Zhejiang University SCIENCE A
ISSN 1009-3095 (Print); ISSN 1862-1775 (Online)
www.zju.edu.cn/jzus; www.springerlink.com
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A general framework for progressive point-sampled geometry

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Received Apr. 17, 2006; revision accepted Apr. 29, 2006

Abstract: Recently unstructured dense point sets have become a new representation of geometric shapes. In this paper we introduce a novel framework within which several usable error metrics are analyzed and the most basic properties of the progressive point-sampled geometry are characterized. Another distinct feature of the proposed framework is its compatibility with most previously proposed surface inference engines. Given the proposed framework, the performances of four representative well-reputed engines are studied and compared.

Key words: Progressive model, Point-sample geometry, Geometric distance, Error measure, Shape representation
doi:10.1631/jzus.2006.A1201 **Document code:** A **CLC number:** TP39

INTRODUCTION

With the recent advances in 3D digital photography and scanning technology, unstructured dense point sets sampled from the surfaces of physical objects have become a new representation of geometric shapes. The shape thus specified is usually referred to as point-sampled geometry (Amenta and Kil, 2004; Liu *et al*., 2004; Pauly *et al*., 2003; Zwicker *et al*., 2002), in which two key techniques are involved:

(1) The raw point data P is discrete in nature while the physical objects to be described must have connected volumes bounded by continuous surfaces. Then, how to infer a continuous surface representation $S(P)$ from P (Fig.1a)?

(2) To achieve high geometric fidelity, the point data output from physically sampling processes usually consists of a large number of points. Then, given any suitable surface inference engine $S(P)$, how to simplify the original point data or convert it into a progressive representation that is often desired by downstream graphics and visualization applications (Fig.1b)?

In this paper we propose a simple and powerful framework that works with most previously proposed surface inference engines and outputs a progressive representation of point-sampled geometry. We first develop a promising progressive model and identify the inherent problems to be solved in Section 2. In Section 3, we present the proposed framework, analyze several usable error metrics and characterize the most basic properties of the progressive point-sampled geometry. The efficiency of any particular surface inference engine depends to a large extent on the type of data involved. Given the proposed framework, empirical studies are conducted and presented in Section 4 to evaluate the efficiency of four well-reputed surface inference engines. Finally our concluding remarks are presented in Section 5.

PROGRESSIVE MODEL AND PROBLEM SPECIFICATION

In real world scenarios, sample point data often suffer from two flaws: noise and undersampling. In our study (Liu, 2003), instead of solving all problems once, we stratify the problems in point-sampled geometry: here we consider the given point data P to be unstructured, free of noise and sufficiently sampled. Each point $p_i \in P$ is associated with a unit normal vector $\boldsymbol{n}_i$.

Let $P' \subseteq P$. Given any well-defined surface inference engine, two surfaces $S(P)$ and $S(P')$ can be inferred. To measure the difference between $S(P)$ and $S(P')$, we use an error metric $d(,)$ induced from an error norm $\|\cdot\|$ (to be defined), such that $d(S(P), S(P'))=\|S(P)-S(P')\|$. Denote the cardinality of a set X as $\#X$. We say, a best approximation to P of order m, subject to the norm $\|\cdot\|$, is a subset $P' \subseteq P$ which satisfies

$$P' = \arg\min_{P''}\{\| S(P) - S(P'') \| : \forall P'' \subseteq P, \# P'' = m\}.$$

A progressive sequence of P is then an ordered set of subsets $\{P_0, P_1, \ldots, P\}$ with orders $\{\#P_0, \#P_0+1, \ldots, \#P\}$, each of which is the best approximation to P with the specified order.

In practice, finding a truly progressive sequence of a large set P by exhaustive searching is unacceptable. Therefore, heuristics have to be used. In (Pauly *et al*., 2002), three heuristics, i.e., clustering, iterative simplification and particle simulation, are analyzed and quantitatively compared. In this work, we propose a progressive model with the following greedy heuristic.

Given a norm $\|\cdot\|$, a point set P and a subset P', a point $p_k \in P'$ is called the finest detail point in P' if it satisfies

$$p_k = \arg\min_{p_i}\{\| S(P) - S(P' \setminus p_i) \|, \forall p_i \in P'\}.$$

We start from the original set P, initialize $P'=P$ and recursively extract the finest detail points d_i from P'. Let the process be terminated at a certain subset P_0. During the process, a sorted set $D=\{\ldots, d_i, d_{i+1}, \ldots\}$ of finest detail points is readily obtained. Then P is decomposed into two parts (P_0, D) where P_0 encapsulates a basic shape of P and D encodes the detail parts of $S(P)$ point by point. A distinct feature of this greedy strategy is, while (P_0, D) offers a richer progressive representation of $S(P)$, both representations

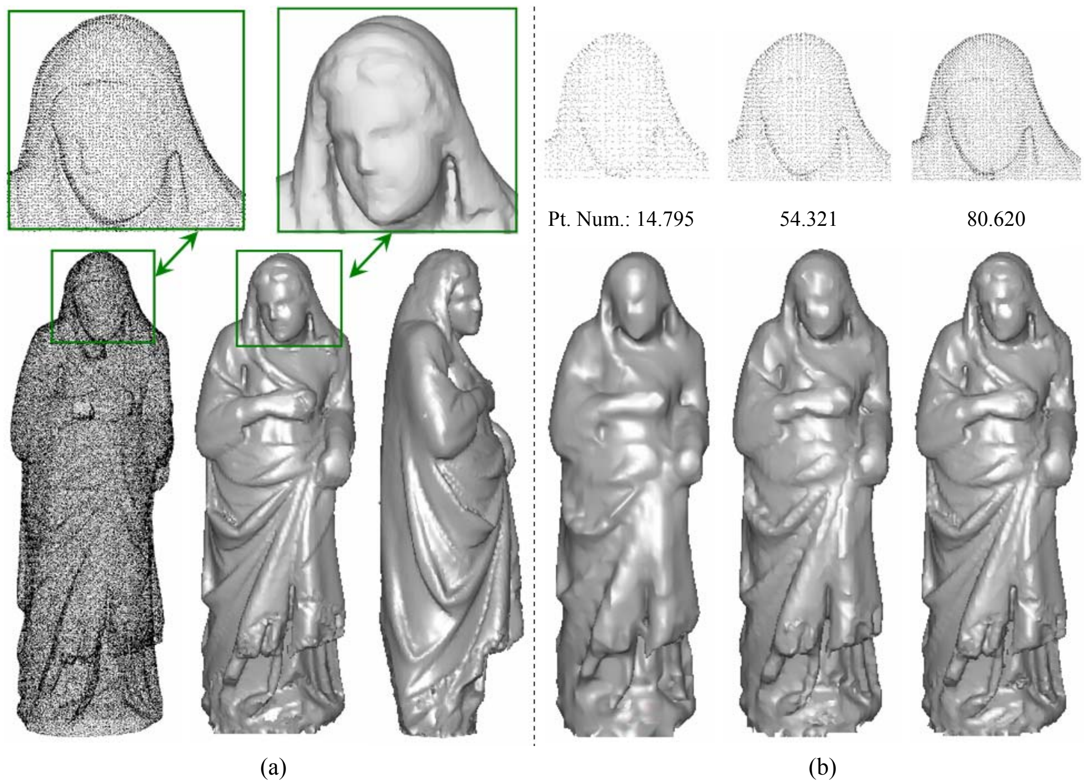

Fig.1 Progressive point-sampled geometry; the original Virgin Mary data are stored in external memory and is preprocessed by applying the dual Hermite downsampling scheme in (Liu *et al*., 2003) to fit in main memory. (a) A continuous surface inferred from 109408 ponits; (b) Progressive refinement

(P_0,D) and P have the same storage since D is just a re-ordering of $P\backslash P_0$. Now the questions are

(1) What is the general form of the error norm $\|\cdot\|$ for progressive point-sampled geometry?

(2) What are the basic conditions that should be satisfied by the proposed norm $\|\cdot\|$?

(3) What are the basic geometric characteristics of the proposed norm $\|\cdot\|$?

FRAMEWORK

To measure the difference between two point-sampled surfaces, a widely accepted metric [e.g., in (Pauly *et al.*, 2002)] is to up-sample the two surfaces and measure the difference by the two-sided Hausdorff distance. In our setting, since the original point data P is free of noise, dense, and sufficiently sampled, we expect the engine S to interpolate P as well as any $P'\subseteq P$; accordingly we propose to measure the difference with the following general q-norm:

$$\| S(P)-S(P') \|_{q\geq 1}=\left(\sum_{p_i\in P\backslash P'} | dist(p_i,S(P'))|^q\right)^{1/q}, \quad (1)$$

where function $dist(p_i,S(P'))$ reports an error distance measure between a point p_i and the surface $S(P')$. If the engine S approximates P', we simply replace the term $p_i\in P\backslash P'$ by $p_i\in P$ in Eq.(1). To study the basic properties inherent in this norm, in the following we consider different types of surface inference engines.

Surface inference engines

Many engines exist in literature on continuous surface inference from dense point sets; among which we analyze three well-known classes. The first class includes the triangulation techniques (Amenta *et al.*, 1998; Edelsbrunner and Mucke, 1994; Liu and Yuen, 2003) that locally connect the data points into a globally structured form—triangle meshes. Two major characteristics are inherent in the triangulation techniques. First, triangle meshes can only accommodate piecewise linear, i.e., C^0-continuous, surfaces. Second, triangulation techniques interpolate point set P and thus, only work for noise-free data and require strict sampling criteria. There are several possible formalizations of the sampling criteria among which two representatives are proposed in (Amenta and Bern, 1999; Cheng *et al.*, 2004).

The second class fits parametric surfaces to the sample points. NURBS surfaces are most commonly used. Either interpolation or approximation can be realized by parametric surface fitting. So far parametric surfaces are best suited for describing shapes with prescribed topological type, mostly homeomorphic to an open disc (Weiss *et al.*, 2002). This gives a very serious limitation to shape that a single parametric patch can model. For shapes with complex topological types, automatic surface fitting with tight tolerances is a difficult task and user interaction is often required to setup a net of reasonable patches (that are usually rectangular) with mounting which needs to maintain continuity across the patch boundaries.

The third class describes point-sampled geometry as level sets of a scalar field defined in $\mathbb{R}^d$. The scalar field can be determined by a differentiable function $F:\mathbb{R}^d\rightarrow\mathbb{R}$ and the level set of F corresponding to a real value $c\in\mathbb{R}$ is the set of points $\{p\in\mathbb{R}^d|F(p)=c\}$. If $d=3$ (resp. 2), the level set is known as a level surface (resp. level curve). In graphics and vision applications, F is usually restricted to algebraic functions and c a regular value of F, i.e., the gradient vector does not vanish at all points of $F^{-1}(c)$. Since the scalar field representation puts no restrictions on the shape's topological type (ref. Figs.1a and 2a), it has been widely used for modeling shapes with both complex topology and rich geometric details. State-of-the-art level set models in graphics and vision applications are motivated by the latest results in approximation theory that takes moving least squares (MLS) methods (Levin, 1998; 2003) and radial basis function (RBF) methods (Lazzaro and Montefusco, 2002; Wendland, 2005) into consideration.

In this work, since we are particularly interested in graphics and vision applications with complex geometric models, in the following, we concentrate on the level set models.

Algebraic vs geometric error distance functions

To use the general norm in Eq.(1), we need to define the function $dist(p_i,S(P'))$ which reports an error distance measure between a point p_i and the surface $S(P')$. One natural choice of the error distance is the shortest Euclidean distance between point p_i and the surface $S(P')$. In the context of level set models, this distance is known as the geometric distance (Sullivan *et al*., 1994). In general, the true geometric distance cannot be computed in closed form so that in practice, the geometric distance is found by solving a nonlinear least squares estimation problem that usually requires the closest point in $S(P')$ to be computed iteratively for every query point p_i. Standard methods such as Levenberg-Marquardt method and variants of Newton's method would be appropriate.

Computationally, it is often more desirable to use the algebraic distance defined as the scalar field value $F(p_i)$ at p_i, where the continuous level surface M is inferred by $F(p)=0$, $\forall p\in M$. Computing algebraic distance is usually fast. However, whether linear or nonlinear, it is well known that the evaluation based on the algebraic distance introduces serious bias since it does not reflect the geometric relationship between the data point p_i and the surface $S(P')$ (Sullivan *et al*., 1994; Taubin, 1991). Our experimental results shown in Fig.2b also demonstrate this characteristic.

In the presented framework, we propose a fast and practical solution that uses a first order approximation to compute the geometric distance. In our setting, each point $p_i\in P$ is associated with a normal vector $\boldsymbol{n}_i$. Given that the data P is uniform and sufficiently sampled, we define a first order approximation of the geometric distance from a query point $x\in\mathbb{R}^3$ to the level surface $S(P)$ as follows.

We find the nearest neighbor $p_{\mathrm{nn}(x)}$ in P for any inquiry point $x\in\mathbb{R}^3$. Then the first order approximation of the geometric distance is defined as

$$geom\_dist(x, S(P)) = (x - p_{\mathrm{nn}(x)}) \cdot n_{\mathrm{nn}(x)}. \quad (2)$$

See Fig.3 for an illustration. To use the above geometric distance approximation, for any progressive approximation $S(P')$, $P'\subseteq P$, we need to maintain a uniform and sufficient sampling of $S(P')$. The upsampling scheme with the uniform particle simulation proposed in (Pauly *et al*., 2002) can be applied here.

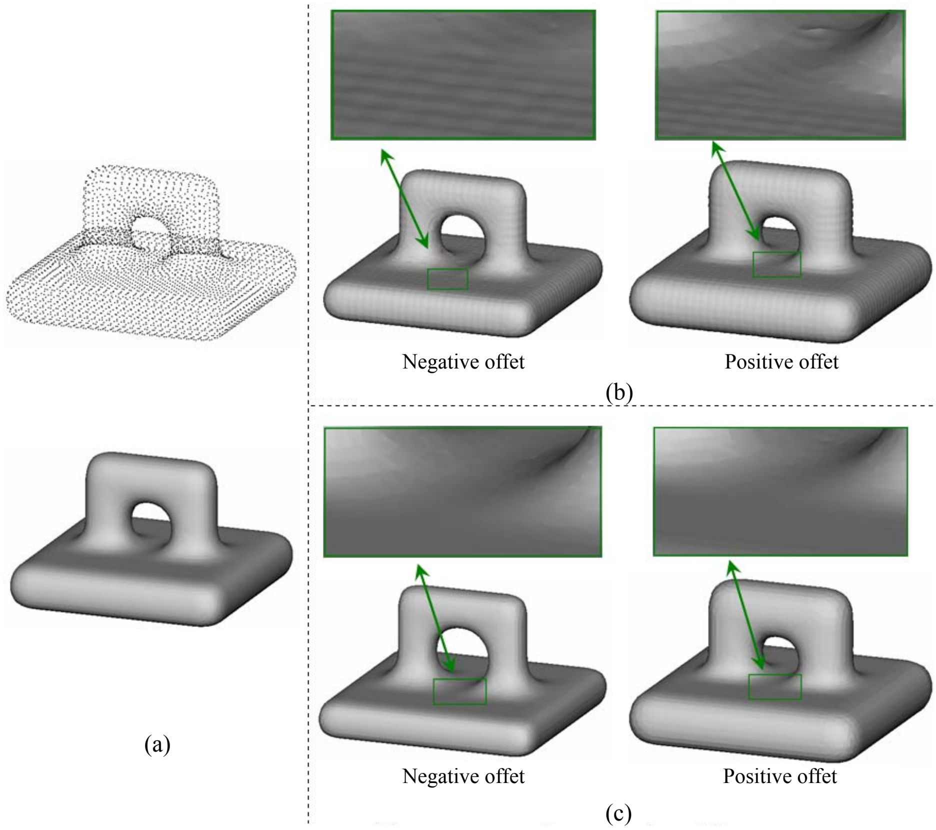

Fig.2 Comparison of algebraic and geometric distances. (a) A continuous surface interred from a set of 3600 points by level set models; (b) Offset surfaces generation by using algebraic distance; (c) offset surfaces generation by using geometric distance

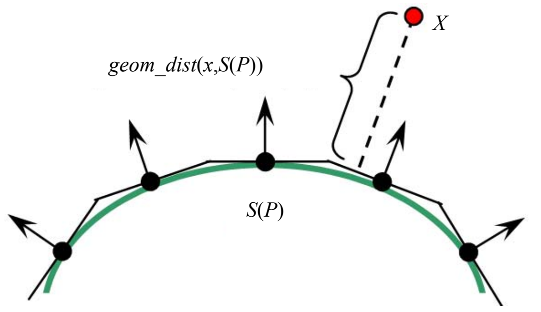

Fig.3 Computation of the first order approximation of geometric distance in $\mathbb{R}^2$

Basic properties of error norm (1)

Definition 1 A point-sampled shape space of order *n* is a set *P* of *n* sample points. Given a surface inference engine *S*, a shape space is normed if for any subset (or subspace) $P'\subseteq P$ there is a real number designated by $\|P'\|_q$, $q\geq1$, where

$$\|P'\|_q=\|S(P)-S(P')\|_q =\left(\sum_{p_i\in P\backslash P'}|geom\_dist(p_i,S(P'))|^q\right)^{1/q}. \quad (3)$$

Definition 2 Given a normed shape space *P*, a surface inference engine is called advanced if it satisfies the strong triangle inequality

$$\|P'\cup P''\|_q<\min\{\|P'\|_q,\|P''\|_q\}, \quad (4)$$

where $P'\neq P''$ are two arbitrarily nonempty subsets of *P*.

In the above definition, the operator "∪" behaves like "+" in the vector space. This definition makes sense since intuitively $P'\cup P''$ contributes more points and thus should infer a better approximate shape than the shape inferred by either P' or P''. The normed shape space so defined satisfies the following algebraic properties:

Lemma 1 Let *P* be an arbitrary normed shape space associated with an advanced surface inference engine. The norm defined in Eq.(3) satisfies the following properties:

(1) $\|P'\|_q\geq0$ (positivity)

(2) $\|P'\|_q=0$ if $P'=P''$ (semi-difiniteness)

(3) $\|P'\cup P''\|_q\leq\|P'\|_q+\|P''\|_q$ (triangle inequality)

where $P', P''\subseteq P$.

We omit the simple proof of Lemma 1 here. In the following, we show that all the norms defined by Eq.(3) are in a sense equivalent.

Theorem 1 If $\|\cdot\|_\alpha$ and $\|\cdot\|_\beta$, $\alpha, \beta\geq1$, are two arbitrary norms defined on a shape space *P*, then $\forall P'\subseteq P$, we have

$$\frac{1}{\sqrt[\alpha]{m}}\|P'\|_\alpha\leq\|P'\|_\beta\leq\sqrt[\beta]{m}\|P'\|_\alpha,$$

where $m=\#(P\backslash P')$.

Proof If $q=\infty$, then

$$\|P'\|_\infty=\max_{p_i\in P\backslash P'}|geom\_dist(p_i,S(P'))|.$$

We have

$$\|P'\|_\infty=(\|P'\|_\infty^q)^{1/q}\leq\left(\sum_{p_i\in P\backslash P'}|geom\_dist(p_i,S(P'))|^q\right)^{1/q} =\|P'\|_q\leq m^{1/q}\|P'\|_\infty.$$

Thus

$$\|P'\|_\beta\leq m^{1/\beta}\|P'\|_\infty\leq m^{1/\beta}\|P'\|_\alpha,$$

and

$$\|P'\|_\beta\geq\|P'\|_\infty\geq(m^{1/\alpha})^{-1}\|P'\|_\alpha.$$

That completes the proof.

Note that given a fixed $m>1$, if $\alpha, \beta>>1$, then $\sqrt[\alpha]{m}, \sqrt[\beta]{m}\to1$ and the lower and upper bounds in Theorem 1 become tighter.

Implementation issues

Recall that in Section 2, we define a progressive model with greedy heuristic by recursively extracting the finest detail points from a point set *P*. To implement this model, we need to choose an advanced surface inference engine and a norm with a specified constant *q*.

Given a point subset P', the desirable surface inference engine should provide points on surface $S(P')$ that are fast and stable such that the uniform upsampling of $S(P')$ for geometric distance calculation (ref. Eq.(2)) can be achieved quickly. This would require that the surface inference mechanism satisfies a local fitting property: recently proposed MLS engine (Alexa *et al*., 2003) based on Levin's projection operator (Levin, 1998; 2003) and engines (Lazzaro and Montefusco, 2002; Ohtake *et al*., 2003; Renka, 1988) based on the partition of unity method fall into this scope. We emphasize that one distinct feature of the presented framework is its compatibility with all these surface inference engines. Since the performance of different engines depend to a large extent on the type of data involved, among these four well-reputed engines, the best one for a class of selected datasets is found in the next section by empirical studies.

In the proposed progressive model, to algorithmically determine the finest detail point from a subset $P'\subseteq P$, for every point $p_i\in P'$, we calculate the error measure $||P'\backslash p_i||$ with a chosen norm and attach it to p_i as a key value. By sorting P' into a queue with the key values, the finest detail point in P' is readily obtained at the top of the queue with the minimal key value. Due to the local fitting property inherent in the surface inference engine, extracting one point p_i from the queue only locally affects key values in a small number of points that are neighbors of p_i in P'. By reassigning the key values and updating the positions of these points in the queue, the recursive extraction of finest detail points is efficiently done. The only thing left now is to choose a norm with a specified constant q in Eq.(3).

Consider the case of removing one point p from a subset $P'\subseteq P$. To evaluate the norm $||P'\backslash p_i||$, instead of calculating all $\#(P\backslash(P'\backslash p_i))$ items inside the summation of Eq.(3), an efficient way is to update the value $||P'\backslash p_i||$ from $||P'||$. Due to local fitting property, this update can be done efficiently by re-evaluating the geometric distances of a few neighboring points of p in $P\backslash P'$. For this updating, if we choose any norm with $q\neq\infty$, we need to record all the values $|geom\_dist(p_j,S(P'))|$, $\forall p_j\in P\backslash P'$. Since by Theorem 1 all the norms are equivalent in a topological sense, we propose to use $q=\infty$ norm, i.e., $||P'||_\infty=\max_{p_i\in P\backslash P'}|geom\_dist(p_i,S(P'))|$: by using this norm, we only need to record one maximal value $\max_{p_i\in P\backslash P'}|geom\_dist(p_i,S(P'))|$ for $S(P')$.

EMPIRICAL STUDIES OF FOUR SURFACE INFERENCE ENGINES

In the presented studies, we examine the performances of four well-reputed surface inference engines by using the proposed framework with a class of selected point models used in (Liu, 2003) in which four of them shown in Fig.4.

The selected geometric data have the following characteristics:

(1) All the data exhibit complex geometric shape with both complex topological types and geometric fine details;

(2) Many data exhibit many tiny planar regions, e.g., plenty of thin-shell structures accommodated in the Wecker and Jalor models in Fig.4. This type of data frequently arises from solid crash simulation, numerical fluid simulation and scientific visualization. We refer to this type of data as CAD-models;

(3) Apart from CAD-models, the rest of the models exhibit smoothness everywhere with some semantic (curved) features. These semantic features are significant for human perception and are best characterized by high curvature variation. We refer to this type of data as graphical-models.

The surface inference engines under investigation include

(1) A modified quadratic Shepard's method (Renka, 1988);

(2) A compactly supported RBF method (Wendland, 1995);

(3) A modified RBF Shepard's method (Lazzaro and Montefusco, 2002);

(4) A moving least-squares method (Alexa *et al.*, 2003) with corrected Levin's projection operator (Levin, 2003).

Note that by using the proposed framework with engine I, the resulting progressive model is similar to the MPU shape scheme proposed in (Ohtake *et al.*, 2003). The above four engines were implemented using Visual C++ platform with a 3D range tree as a ground data structure to support range queries and k-nearest-neighbors searching (Chávez *et al.*, 2001).

Parameter settings

Each of the studied engines needs to specify several parameters for a given object. For faithful comparison, we keep as many as possible of the parameters at their original settings. For engine I, we use the settings in (Ohtake *et al.*, 2003): (1) by using a range tree for k-nearest-neighbors searching with a fixed k, we can obtain the value r of radius of a bounding sphere for each inquiry point; (2) the support

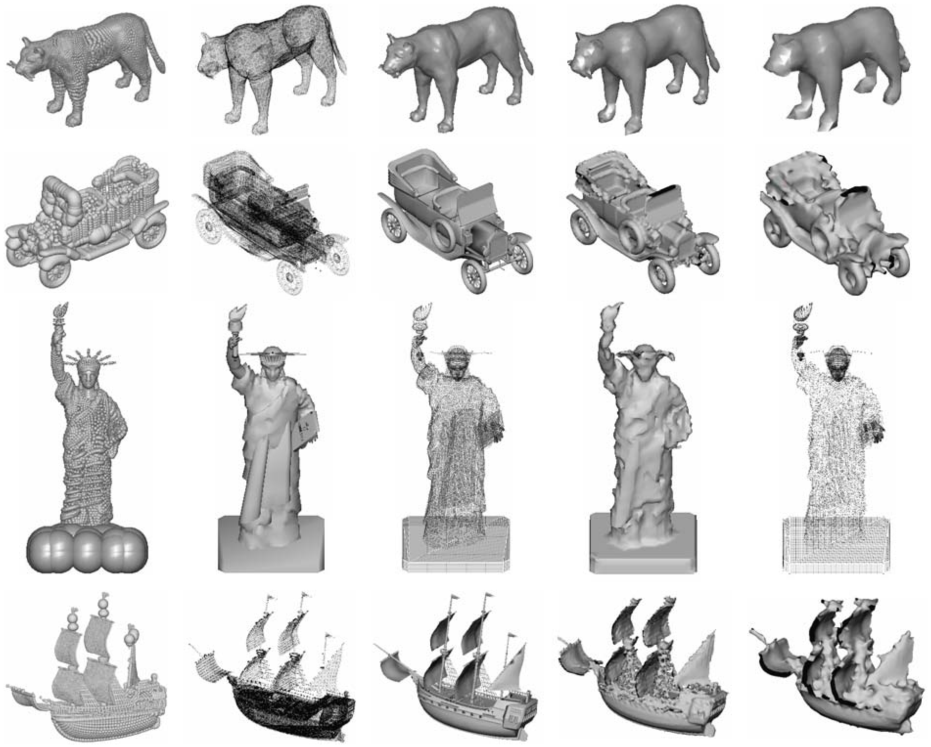

Fig.4 Four typical test models and their representative progressive sequences

radius R of the corresponding weight function is set to be R=1.5r. For RBF engines (including both engines II and III), only providing surface points with scalar field value zero can lead to trivial solutions; thus by following the settings in (Carr *et al.*, 2001), we generate two off-surface points $op_{\pm}=p_i \pm cn_i$ for each sample point p_i with unit normal $\boldsymbol{n}_i$ and c=0.1r. To implement engine IV, let $B=\{x\in\mathbb{R}^3 \; \|x-p_i\|<r_i, \; \forall p_i\in P\}$ be a union of open balls with radius r_i centered at each point $p_i \in P$ to define a tubular neighborhood of P in $\mathbb{R}^3$. To calculate an MLS projection operator $P_{\mathrm{MLS}}:B\rightarrow B$, a local reference domain (a plane if in $\mathbb{R}^3$) is determined by minimizing a non-linear energy functional with a non-negative weight function θ. The widely used weight function $\theta(r)=\mathrm{e}^{-r^2/h^2}$ gives C^∞ continuous MLS surfaces. As suggested in (Alexa *et al.*, 2003), the parameter h is not necessarily global and could be adapted to the local feature size. In our setting, we choose h locally to be the value r_i of radius of bounding sphere for each point p_i.

All the tests shown in this paper were performed on an off-the-shelf PC with 512 MB RAM and a Pentium III processor running at 937 MHz operated by Microsoft Windows XP. The performance of the four engines is evaluated based on the following two criteria.

Time and space efficiency

Fig.5 shows computational time and memory requirements for different surface inference engines. Two notes follows. First, the presented data is obtained by our implementations of different engines which may not be the most optimum for code generation. Second, since the progressive models only need to be generated once, these generations can be performed in an offline mode and therefore, the next criterion on geometric error is more crucial.

Geometric error

Fig.6 shows the quantitative error estimates of four models in Fig.4 by using the proposed first order approximation of geometric distance in Eq.(2) and the error norm $\|\cdot\|_\infty$ in Eq.(3).

As shown by Figs.5 and 6, we reach the following conclusions:

(1) For all the cases, engine II runs the slowest and consumes the largest memory even with an optimal coding for sparse matrix solver, and has relatively inferior reproduction property in terms of

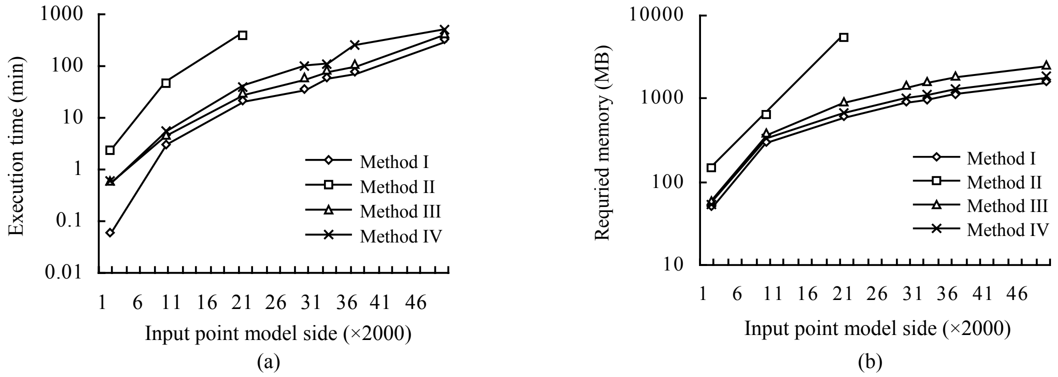

Fig.5 Execution time (a) and memory requirement (b) for the four surface inference engines running with the point models shown in Figs.1, 2 and 4. In these figures, time and memory axes is logarithmic. For testing on large point models, in addition to 512 MB physical memory (RAM), the system also sets up 3.5 GB virtual memory

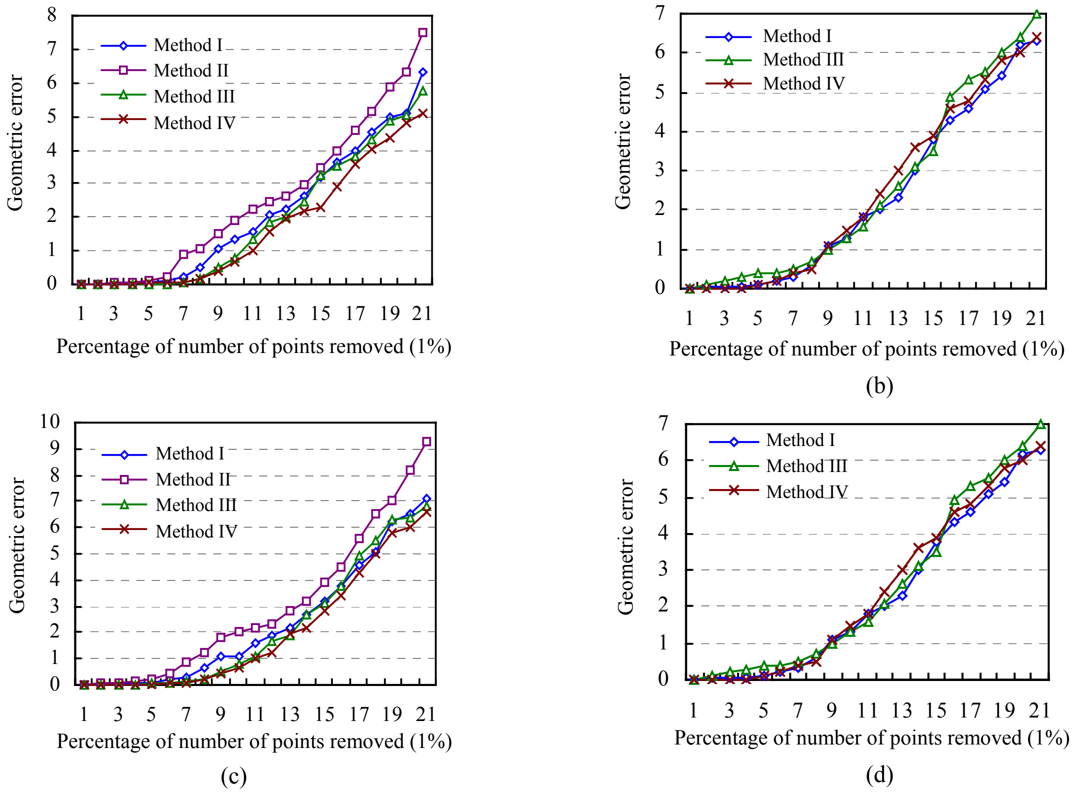

Fig.6 The geometric approximation properties of the four studied engines running with the four models are shown in Fig.4. In all the tests, the geometric error is related to the diagonal length of the cube bounding the model. Due to unacceptable memory requirement, engine II with Wecker and Jalor models are not tested

geometric shape inference. Thus we would not recommend engine II as a candidate engine for progressive point-sampled geometry;

(2) For CAD-models, engine I shows good reproduction property: this can be interpreted as that the quadratic surface works well for approximation of near-planar regions. Engine I also runs the fastest and is easy to implement;

(3) For graphical-models, engines III and IV offer good balance between running time and reproduction properties. In all our tests, engine IV slightly outperforms engine III. This result comes up to our expectation that moving least-squares surfaces should offer good approximation to C^{∞} smooth surfaces.

CONCLUSION

In this paper, a general framework is proposed to address the most basic characteristics inherent in progressive point-sampled geometry. The proposed framework has the following distinct features:

(1) A promising progressive model with developed greedy heuristic;

(2) A general q-norm based on geometric error measure is proposed to quantitatively characterize the approximate shapes;

(3) A first order approximation of geometric distance from a point to a surface is adopted to rapidly obtain a geometric error measure;

(4) Practical implementation issues are addressed with the proposed error norm;

(5) Empirical studies of four well-known surface inference engines are presented and quantitatively compared.

ACKNOWLEDGEMENT

The authors would like to thank the Visual Computing Lab for publishing their Virgin Mary data on the Web http://vcg.isti.cnr.it/.

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